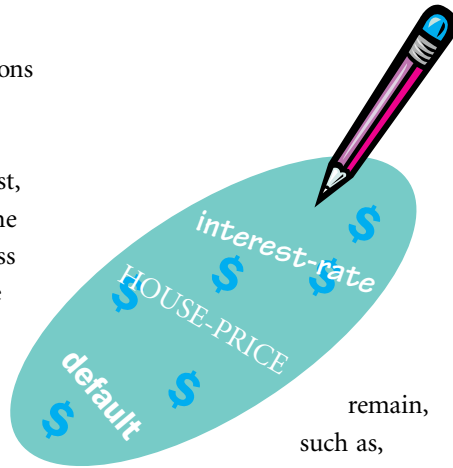


# Drawing an Ellipse Around Risks

GEARING UP TO SURVIVE catastrophic economic conditions is one way for a business to manage its financial risks. If a company can weather the worst, then surely it can withstand the lesser hardships that might pass its way. Through the surrogate of **stress testing**, a company can measure its performance under an infinite number of financial thrashings, such as a large increase in interest rates or a prolonged economic recession.

Companies stress test by building two types of economic models to determine the effects of damaging conditions on their operations. One type of model depicts how the underlying economic factors that best represent a firm's risks would behave when shocked. The other, a cash-flow model, projects the company's performance in response. That still leaves stress-test designers to decide which simulations are severe enough but stop short of arbitrarily harmful scenarios irrelevant to the firm's real-world economic risks.

Studying **joint-probability distributions** can help companies sort through the risks they might face. Little doubt exists that a large interest-rate shock or a prolonged recession creates stress for a company. Yet, other questions



remain, such as, What is the probability of both happening concurrently? When only two economic factors account for most of the risks facing a firm, it becomes possible to draw pictures of the joint-probability distributions that take the shape of an ellipse. By providing a visual presentation depicting the likelihood of the two bad events occurring simultaneously, the **joint-probability ellipse** helps identify the sets of circumstances on which a company should concentrate in its stress testing.

## When Bad Things Happen to Good Companies

Interest rates and house prices pose the greatest economic threats to Freddie Mac, Fannie Mae and most thrifts. To expose a firm's vulnerabilities, a stress test would replicate the movements of the two factors when subjected to economic shocks then project how a company's cash flows would

respond. The Great Depression, for instance, sent both interest rates and house prices tumbling as unemployment soared. The first two variables, however, can move in the same direction or in opposite directions for a total of four general interest-rate and house-price environments. Any number of extreme combinations of these two factors can lead to economic disasters of varying probabilities that produce corporate losses rivaling or exceeding the Great Depression's damage.

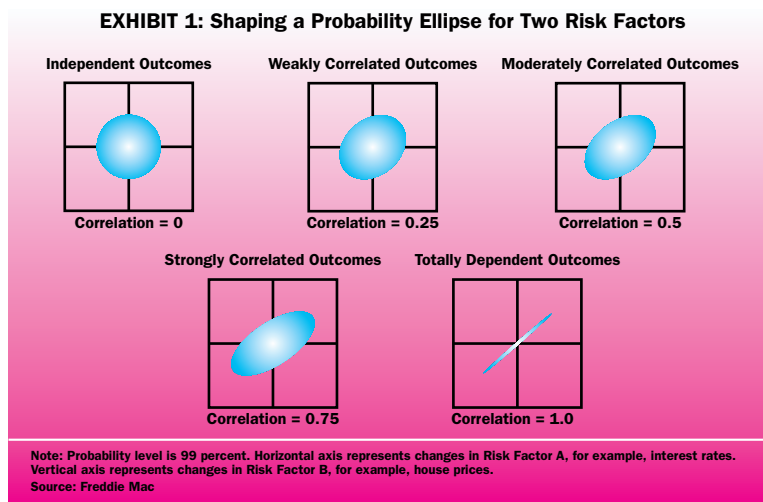
Stress tests do not require elaborate implementation preparations, yet they provide a painless way for a company to seemingly experience events that otherwise could be difficult to endure. Nonetheless, stress tests provide no information about how likely these situations are to occur. To cover its bases, a company theoretically could investigate every conceivable calamity. Still, the firm would need to choose the catastrophes against which to capitalize in the same way that a structural engineer might design a dam to withstand a rare but devastating event, such as a 100-year flood.

**Value-at-risk** models are useful for analyzing the market

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by Donald Solberg and Vassilis P. Lekkas



The strength of the statistical interdependence of two variables, or risk factors, determines the shape of the elliptical probability field, which can range from a circle showing no correlation to a straight line indicating that a change in one factor is accompanied by a highly predictable change in the other. Despite a range of shapes, the hypothetical ellipses shown above share the same 99-percent probability level, which means the points outside each ellipse will occur less than 1 percent of the time.

risks of bank trading-book activities. However, implementation difficulties make this approach less feasible for evaluating nontraded risks such as the default risk that companies like Freddie Mac and Fannie Mae face from their mortgage-backed-security guarantees or their mortgage portfolio holdings. Stress tests, on the other hand, provide a computationally feasible way to integrate traded and nontraded risks, but they do not incorporate the probability assessment that is a key ingredient of value-at-risk calculations.

Ideally, then, a company could combine elements of stress testing and value-at-risk modeling to devise a reasoned means of selecting a limited number of stress-test scenarios that stand the best chance of taking place. A joint-probability distribution as depicted by an ellipse is capable of doing just that. Although stress tests typically are built around specific and severe historical anomalies, this decision-making framework focuses on the interplay between the underlying economic factors. Freddie Mac employs a modified version of this statistical tool to examine interest-rate and house-

price interactions as part of the company's capital-adequacy practices.

The relevance of worst-case testing becomes much greater when a joint-probability distribution identifies the likelihood of two bad events taking place in combination and assesses a company's chances of failure under those circumstances. A company setting an internal capital-adequacy target that limits failure from interest-rate and house-price risks to a 1-percent chance or less, for instance, would construct an ellipse at a 99-percent probability level. The probability feature establishes an elliptical boundary of interest-rate and house-price movements that excludes extremely rare combinations, which, in this case, means those with less than a 1-percent chance of occurring. By imposing a probability screen, the ellipse moves stress testing towards a risk-detection system more closely resembling the value-at-risk measures favored by many large financial institutions.

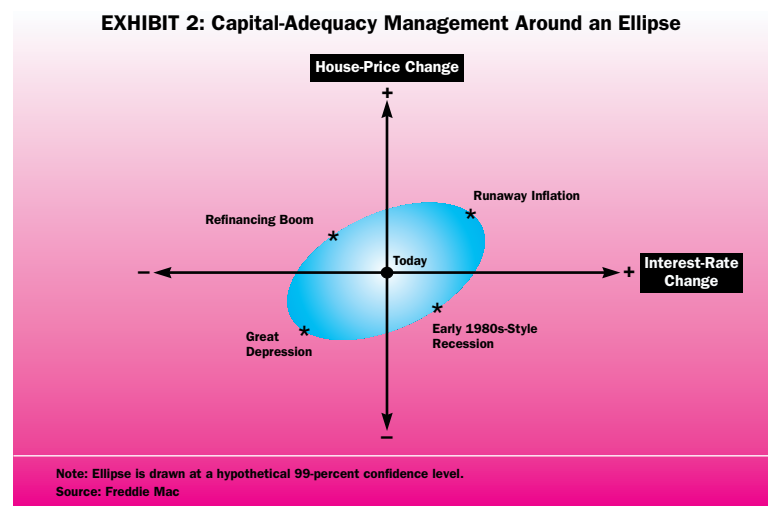
Capital adequacy, as defined by an analysis of joint-probability distributions, consists of holding enough capital to survive the worst of the events encompassed within and along the borders of an ellipse. To quantify the amount of capital that would take, a firm would need to perform a number of the ellipse stress tests. After isolating the most damaging, the company

would set aside enough capital to persevere under those conditions for a pertinent length of time, such as the 10-year span that the Office of Federal Housing Enterprise Oversight (OFHEO) has selected as the basis of the regulatory stress test it is developing.

### Getting Down to Statistics

When plotted out, the pattern of possible risk-factor combinations can take a variety of elliptical shapes, ranging from a circle to something approaching a straight line. The more elongated the shape, the greater the correlation between the two risk factors—that is they tend to move together. If the two risk factors are positively correlated, then the ellipse tilts up towards its northeast quadrant, as do those in *Exhibit 1*. If, however, a change in one risk factor parallels an opposite movement in the other, then the pair are negatively correlated and the ellipse assumes a northwest-to-southeast slant.

Statistically speaking, the two-dimensional ellipse is derived from a three-dimensional form depicting the likely distribution of interactions between a pair of risk factors. Each risk factor contributes a separate dimension, while the probability of each risk combination supplies the third dimension. A way to think about an ellipse is as a contour map of a hill where the longitude and latitude are equivalent to the two risk factors and the elevation is



**Under this ellipse framework, a well-capitalized company would need to withstand the worst of four economic environments—runaway inflation, monetary-policy-induced recession, economic collapse and a refinancing boom—for an extended period of time, perhaps a decade, without going bankrupt.**

equivalent to the probability. Each contour line connects points equal in elevation (probability). The hill consists of a series of ever smaller ellipses, with each ellipse representing a particular contour line or probability level.

If an ellipse is drawn at a 95-percent probability level, points outside the figure depict extreme events that will occur only 5 percent of the time. Risk pairs closer to the center—or, nearer the top of the hill—are more likely to occur on average than those located farther away.

The size of an ellipse depends upon the probability value to which it is drawn. An ellipse containing 97 percent of all outcomes covers a larger surface area than one configured to a 95-percent probability.

### Building an Ellipse with Confidence

Over a long period of time, house prices and interest rates have demonstrated a positive correlation that is moderate in its intensity. For example, growth in house prices might accompany a rise in interest rates when general inflation is driving both trends. If the two events were uncorrelated, changes in interest rates would not track changes in house prices.

Consequently, a firm that is highly concentrated in mortgages may want to test the strength of its capital against a series of situations sufficient to confidently capture 99 percent of possible interest-rate and house-price combinations. An ellipse like the one shown in *Exhibit 2* could help the company identify

an appropriate mix of stress tests to achieve that degree of probability. The horizontal axis of this particular ellipse represents interest-rate changes, and the vertical axis plots movements in house prices. The center point of the ellipse signifies current conditions, such as stable interest rates and, perhaps, a 3-percent annual rate of house-price appreciation.

The ellipse illustrated in Exhibit 2 is used for the purpose of managing capital adequacy. The outermost ellipse point marked in each quadrant tells a story about different economic environments:

- The northeast quadrant depicts runaway inflation that launches house prices and interest rates on an upward trajectory.
- The southeast corner reflects the unusual kind of **disinflationary** conditions that resulted in the early 1980s from the country's tight monetary policy. The shrinking money supply forced interest rates higher, sending the economy into recession and slowing inflation. House-price appreciation weakened during the economic contraction and, in some cases, property values actually declined.

*Some of the extreme events along the edge of a probability ellipse may not be stressful. Instead, they could amount to lucrative opportunities.*

- The southwest portion portrays the kind of **deflationary** conditions present during the Great Depression, when both house prices and interest rates fell dramatically as the economy collapsed.
- The northwest quadrant brings together the ingredients necessary to ignite a refinancing boom. Falling interest rates and rising home values would give borrowers both the incentive and the ability to prepay their mortgages.

#### **Maintaining a Constant Vigil**

A prudent company will re-estimate probability relationships and redraw its ellipse on a regular basis. Over time, the mix of house prices and interest rates can and does evolve, leading to economic shocks that differ in

type or degree. Similarly, the stress tests that at first seemed to reflect a firm's risk exposure may require commensurate updating to accurately deliver the kind and strength of shock needed to provide the lesson sought.

An ellipse is silent as to the magnitude of a firm's potential losses under any of the economic scenarios encompassed by it. In fact, some of the extreme events along the edge of a probability ellipse may not be stressful at all. Instead, they could amount to lucrative opportunities.

A company pursuing complicated and exotic business strategies may find that some less extreme but more common events inside the ellipse could conspire to create losses higher than those occurring along its lip. In the interests of vigilance, a firm following that kind of game plan could elect to divide the ellipse into a grid and perform the stress test dictated by the midpoint of each cell. By constantly monitoring a wide variety of points like this, a company can stay attuned to the risk combinations that pose the most common, most extreme economic threats to its well-being.

An ongoing re-examination of the premises that underlie an ellipse and its stress-test components provides a firm with a more accurate picture of its risk exposure that is central to determining internal capital-adequacy strategies. **SMM**